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Predicting the Effects of Surface Roughness on Laminar-Turbulent Transition for Axisymmetric Bodies

M. J. Casarella*
Catholic University, Washington, D.C.
and
J. M. Niedzwecki†
Texas A & M University, College Station, Texas

A procedure for assessing the effects of uniformly distributed surface roughness on the location of laminar-turbulent transition is presented. A dimensionless roughness Reynolds number $\bar{R}_K$ is defined which allows one to isolate the effects of body shape on causing premature transition due to roughness. A comparative evaluation of the sensitivity of body shape to tripping the flow can be made by examining the variation of $\bar{R}_K$ along the surface from a class of geometric shapes. The results demonstrate that secondary effects such as roughness can play a critical role in the location of the laminar-turbulent transition. Extensive numerical results were obtained for elliptical forebodies. Their sensitivity to roughness and the determination of critical roughness heights over a wide range of body Reynolds numbers are presented. For these shapes, it was found that the critical roughness height could be expressed as a function of the body Reynolds number.

Nomenclature

D = reference length, body diameter
f = dimensionless stream function in transformed coordinates
K = roughness height
H = shape factor, $H=\delta^*/\theta$
m = pressure gradient parameter for axisymmetric flows
P = dimensionless parameter for axisymmetric flows
r = radial coordinate
R_D = body Reynolds number
$\bar{R}_K$ = new roughness Reynolds number
R_K = roughness Reynolds number
R_δ^* = displacement thickness Reynolds number
s = coordinate along body surface
x = coordinate along the centroidal axis
y = vertical coordinate
u = horizontal component of velocity
v = vertical component of velocity
α = body shape constant
β = pressure gradient parameter
δ^* = displacement thickness
Δ = displacement thickness in transformed coordinates
η = transformed coordinate vertical component
θ = momentum thickness
μ = body shape constant
ν = kinematic viscosity
ν_k = momentum diffusivity
ζ = transformed coordinate horizontal component
ψ = dimensional stream function

Subscripts

crit = critical value
e = edge of boundary layer
inst = instability
max = maximum
tran = transition value
w = evaluated at the wall
∞ = evaluated at infinity

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*Professor of Mechanical Engineering. Research Scientist, David Taylor Naval Ship Research and Development Center.

†Assistant Professor of Ocean and Civil Engineering.

Introduction

THE accurate prediction of the laminar-turbulent boundary layer characteristics for steady flows around submerged bodies, blades of turbomachinery, and aerodynamic surfaces is performed quite regularly. The recent trend has been to introduce local secondary effects into the computational algorithm. Effects such as heating, porous suction, compliant wall, and surface protuberances are studies to see how they influence the features of the boundary layer. This investigation focuses upon the implications of introducing uniformly distributed surface roughness into the boundary-layer computations for external flow problems.

Currently, in computational fluid mechanics, there seem to be two main thrusts in surface roughness research. The first effort, which is the subject of this paper, is to study the influence of surface roughness on the laminar-turbulent transition. The second thrust is directed at providing a model capable of simulating the boundary-layer development in the turbulent flow regimes.[1,2] This investigation is concerned with the sensitivity of laminar velocity profiles to various roughness heights as well as their ability to forestall the onset of transition.

The physical features of the transition process on a smooth surface have been carefully examined in the laboratory.[3-5] The transition mechanism can be envisioned as a process in which random turbulent bursts or spots begin to appear in the laminar flow immediately downstream of point A in Fig. 1. Simultaneously, turbulent bursts begin appearing downstream with increasing frequency until at point B the flow is fully turbulent. When the boundary layer is tripped by localized or distributed roughness the transition region collapses into an almost abrupt laminar-turbulent process. Typically, localized or isolated roughness elements will trip the boundary layer upstream of the natural transition point. At the location of these isolated roughness elements, wedge-shaped regions of turbulence appear around the circumferential surface. These wedge-shaped regions then coalesce or blend with the natural transition downstream.

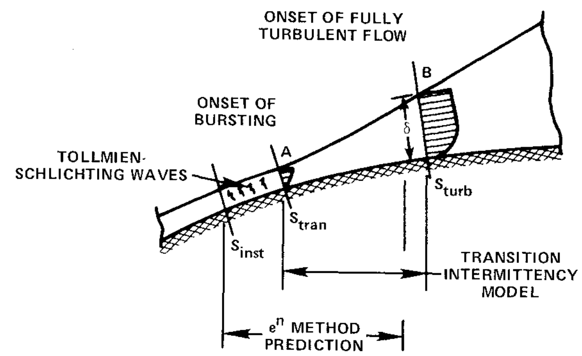

Fig. 1 Details of natural transition on smooth surface.

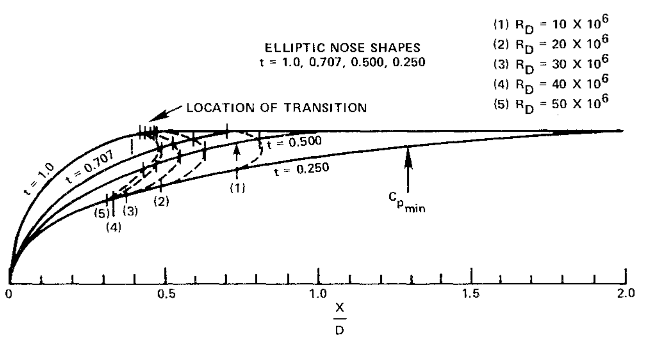

Fig. 2 Effects of nose shape and body Reynolds number on location of transition (see Ref. 15).

The transition process is caused by "instability" of the flow, that is, the tendency for small disturbances (due to noise, mechanical vibrations, surface roughness, freestream flow disturbances) to be amplified into the burst eruption process. The classical method for predicting the instability threshold for wave growth is based upon linear stability theory which attempts to predict whether these disturbances, inherent in all flows, will grow or decay during the downstream development of the laminar boundary layer.

Prediction of the complete transition process is far less amenable to rigorous analysis since the process is governed by external disturbances which interact with the boundary layer. Thus the characterization of the unsteady and steady disturbances external to the attached laminar flow as well as the surface features of the wall are critical.

The most widely used method for predicting transition (often defined as the location for the onset of fully turbulent flow) is the e^n method. The advantages and disadvantages of this disturbance amplification method are discussed in recent papers by Mack[6] and Wazzan, Gazley, and Smith.[7] They state that despite much criticism of the method, the approach is particularly well-suited to design studies where an optimum body shape is sought that will give the highest possible transition Reynolds numbers. The e^n method of predicting transition requires accurate computation of the potential flowfield and laminar boundary-layer properties and the evaluation of Orr-Sommerfeld eigenfunction. An unpublished efficient computer program has recently been developed by von Kerczek and Graves of the David W. Taylor Naval Ship Research and Development Center (DTNSRDC) (report with limited distribution). Some typical results for elliptical nose shapes based upon e^n prediction for the location of transition are shown in Fig. 2. These results show the effects of both geometrical shape and body Reynolds number on the location of laminar-turbulent transition.

Since this investigation is solely concerned with the effects of only one type of disturbance-generating source (distributed surface roughness), a simplified method will be used for predicting the location of the onset of burst eruption. The instability condition and the width of transition will be predicted using a semiempirical approach. This method maintains all of the qualitative features of the e^n method while minimizing extensive computations during the preliminary design stages. The underlying basis for this simplified method is that the instability threshold is envisioned as the condition under which the downstream growth of the boundary layer, denoted by the displacement thickness Reynolds number R_{δ^*}, exceeds a critical value $(R_{\delta^*})_{crit}$ and that the linear stability program is essentially performing this computation as rigorously and accurately as possible.

This critical value of the displacement Reynolds number $(R_{\delta^*})_{crit}$ is obtained at each point along the body from an assumed universal curve of the displacement Reynolds number $(R_{\delta^*})_{crit}$ vs shape factor H. The shape factor H is the local value computed at that location by the laminar boundary-layer computations. These curve-fitted data for $(R_{\delta^*})_{crit}$ vs H have been generated from the Falkner-Skan boundary-layer profiles over a range of favorable pressure-gradient conditions. Table 1 lists some typical data.

The inherent assumption being made is that the actual nonsimilar axisymmetric boundary-layer profile computed at each point along these well-behaved elliptical forebodies are Falkner-Skan type profiles with β = constant. Since the pressure-gradient parameter β varies considerably with body shapes, this assumption is not satisfied. However, by comparing the computed velocity profiles and boundary-layer properties at selected positions in the favorable pressure-gradient region (where transition is likely to occur at high Reynolds numbers) with the published similarity profiles, it indicates less than 5% error in the displacement thickness and 8% error in the shear stress for the smooth wall results. This is an acceptable error for purposes of this investigation in view of the relative accuracy of the e^n method.

Surface Roughness Effects on Transition

The effect of roughness on the onset of transition from laminar to turbulent flow has been extensively investigated. In order to interpret these research studies for practical applications, one must clearly distinguish between the several categories of roughness. This includes single isolated two-dimensional protrusions, regular well-defined grooved or wave patterns, and irregular random distributed roughness. Each type can affect the transition process differently. Furthermore, the characteristic parameters which quantify the degree of roughness are different for each of these classifications.

Dryden[8] and Smith and Clutter[9] made noteworthy contributions during the early stages of research on the effects of roughness. Later, Tani presented his data along with Dryden and others concerning the global aspects of roughness on transition. Klebanoff and Tidstrom[10] concentrated on obtaining a more basic understanding of the transition process. They presented detailed measurement on the mean velocity profiles and disturbance velocity spectra downstream of two-dimensional cylindrical roughness placed on the surface of the plate. The diameter of these cylinders was much smaller than the laminar boundary-layer thickness. This study identified the mechanism for which a single two-dimensional roughness

Table 1 Stability parameters for Falkner-Skan profiles (from Ref. 17)

H	$(R_{\delta^*})_{crit}$
2.296	7680
2.325	6230
2.361	4550
2.410	2830
2.480	1380
2.591	520
2.801	199

caused distortions in the mean flow and it was subsequently used by Kosecoff, Ko, and Merkle[11] to model distributed roughness. In this model, it is assumed that unsteady velocity fluctuations generated by these distributed roughness elements near the wall, serve as a source of augmented momentum transfer and that these sources can be modeled by a local eddy diffusivity in the laminar boundary-layer equations. In effect, the authors' state that a "turbulent roughness layer" with a thickness of the effective roughness height is assumed to be imbedded within the ordinary laminar boundary layer.[11,12]

The phenomenological model proposed by their work was used in this investigation to generate "rough wall" laminar profiles. The stability of these profiles was then predicted using the simplified method.

Governing Equations

The governing equations within the boundary layer are the continuity and momentum equations. In terms of an axisymmetric coordinate system, the governing equations for laminar flow of an incompressible fluid are

$$\frac{\partial}{\partial s}(ru)+\frac{\partial}{\partial y}(rv)=0 \qquad (1)$$

$$u\frac{\partial u}{\partial s}+v\frac{\partial u}{\partial y}=u_e\frac{\mathrm{d}u_e}{\mathrm{d}s}+\frac{1}{r}\left[r(\nu+\nu_K)\frac{\partial u}{\partial y}\right] \qquad (2)$$

where ν_K is momentum diffusivity due to surface roughness.[11,12]

The corresponding boundary conditions are

$$u(s,0)=0 \qquad (3a)$$

$$v(s,0)=0 \qquad (3b)$$

$$\lim_{y\to\infty} u(s,y)=u_e(s) \qquad (3c)$$

Although the numerical computations could be performed in the physical space (s,y) it is more advantageous from a numerical computation point of view to use a transformed space (ξ,η). Both spaces are related by a generalization of the Mangler and Levy-Lees transformations and shall be referred to as the "generalized Mangler transformation."[13-15] This transformation has the form

$$\xi=\mu^2 R_D m^2 (r/D)^2/2 \qquad (4)$$

$$\eta=(u_e/u_\infty)\sqrt{R_D}/m\cdot(y/D) \qquad (5)$$

where the following dimensionless quantities have been introduced

$$R_D\equiv u_\infty D/\nu \qquad (6)$$

$$m\equiv\sqrt{(u_e/u_\infty)(s/D)/P^2} \qquad (7)$$

$$P^2\equiv(u_e/u_\infty)(r/D)^2(s/D)\bigg/\left[2\int_0^{(s/D)}(u_e/u_\infty)(r/D)^2\mathrm{d}s\right] \qquad (8)$$

and the characteristic length D and the freestream velocity u_∞ have been used to obtain the dimensionless quantities. Upon the introduction of the generalized Mangler transformation and a dimensionless stream function of the form

$$f(\xi,\eta)=\psi(s,y)/(D\sqrt{2\xi}) \qquad (9)$$

one obtains the governing equation and boundary conditions in transformed space. Specifically, the governing equation is

$$(bf'')'+ff''+\beta[1-(f')^2]=2\xi\left(f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi}\right) \qquad (10)$$

where the trasnformed boundary conditions are

$$f(\xi,0)=0 \qquad (11a)$$

$$f'(\xi,0)=0 \qquad (11b)$$

$$\lim_{\eta\to\infty} f'(\xi,\eta)=1 \qquad (11c)$$

The following definitions have been used to obtain Eq. (10)

$$(\,)'\equiv\frac{\partial}{\partial\eta}(\,) \qquad f'=\frac{u}{u_e} \qquad (12a)$$

$$b=1+\nu_K' \qquad \nu_K'=\nu_K/\nu \qquad (12b)$$

$$\beta=M/2P^2 \qquad M=\frac{2(s/D)}{u_e/u_\infty}\frac{\mathrm{d}(u_e/u_\infty)}{\mathrm{d}(s/D)} \qquad (12c)$$

A general discussion of the solution procedure and the techniques used to obtain the numerical results can be found in Refs. 14 and 15.

Numerical Results

Estimation of Critical Roughness Reynolds Number $(R_K)_{\mathrm{crit}}$

The distinction between the computer results obtained for laminar profiles on a smooth surface and those obtained on a rough wall are directly attributable to the roughness eddy viscosity ν_K in the analytical model. The parameter governing the magnitude of this term is the roughness Reynolds number R_K. This phenomenological model for roughness, developed by Merkle and his colleagues,[12] is written in the transformed variables as

$$\nu_K'=0.094\,R_K[1.0-\exp(-R_K/40)]\exp(\eta/\eta_K)^2 \qquad (13)$$

where

$$R_K=u_K K/\nu \qquad (14)$$

Note that K is the dimensional roughness height and u_K is the value of the laminar velocity profile at the roughness height.

By inspecting Eq. (12b) one can conclude that the magnitude of

$$(\nu_K')_{\max}=0.094R_K[1.0-\exp(-R_K/40)] \qquad (15)$$

relative to one governs its effect on the numerical solution of the equation at each position along the surface of the body. Some of the computer results using this roughness model on an elliptical forebody are tabulated in Table 2. The data show the effect of increasing roughness on the profiles at a fixed position on the body and prescribed R_D. Relatively small values of R_K ($R_K\leq 10$) have a significant effect on the wall shear stress and shape factor H but no appreciable effect on the general shape of the velocity profile.

The essential question is "what is the critical value of roughness $(R_K)_{\mathrm{crit}}$ for which the laminar profile is altered to the extent that it will cause transition to move upstream?" The simplified method of predicting transition as previously discussed, is based on the condition when $R_{\delta^*}(s)\geq(R_{\delta^*})_{\mathrm{crit}}$ where $(R_{\delta^*})_{\mathrm{crit}}$ is solely dependent upon the local shape factor

Table 2 Effects of increasing roughness on laminar profiles on elliptical forebody ($t=0.500$, fixed position $s/D=0.238$, $R_D=50\times10^6$)

$K/D\times10^5$	η_K	f'_K	f''_w	H	R_K
0.13888	0.024	0.019	0.7928	2.370	1.05
0.27777	0.048	0.035	0.7665	2.373	4.10
0.55555	0.094	0.058	0.5918	2.426	13.36
0.83333	0.141	0.067	0.4232	2.533	22.86
1.11111	0.188	0.070	0.3171	2.666	31.78
1.38888	0.236	0.071	0.2511	2.806	40.38
2.7777	0.471	0.074	0.1259	3.500	83.89

$H(s)$ and is obtained from curve-fitted data similar to that given in Table 1. Consequently, the effects of roughness on H is of primary concern. The numerical results shown in Table 3 for an elliptical forebody indicate with some consistency that, for $(R_K)_{\mathrm{crit}}\simeq4.0$, H changes by an amount detectable for stability computations. These results cover a range of body Reynolds numbers R_D and locations on the surface s/D for which natural transition is likely to occur.

For large values of roughness $R_K\geq25$ where $(K/\delta^*)\geq0.15$ and $(K/\theta)\geq0.3$, the results obtained using the simplified method of predicting transition should be cautiously interpreted. For large roughness where the laminar boundary layer can be tripped a noticeable distance upstream of the location for natural transition, the physical interpretation of the transition region, including the instability threshold and burst eruption region, has not been resolved. Some preliminary unpublished results at the DTNSRDC indicate that an abrupt transition occurs and that the width of this narrow region is independent of Reynolds number. In view of the lack of understanding of the breakdown mechanism for a tripped boundary layer, the more rigorous e^n method for predicting transition is also subject to questionable application.

Sensitivity of Various Body Shapes

It has been widely established experimentally that the roughness Reynolds number is the dimensionless quantity that most appropriately reflects the sensitivity of laminar flow to various types of surface roughness. First, let us consider the idealized case of uniformly distributed roughness where K has a fixed value over the complete surface of the forebody and $u_K(s)$ is the local velocity at a distance K above the surface. Then, the question of tripping the flow by localized roughness at a prescribed location will be examined. Let us begin by considering uniformly distributed roughness. Since,

$$R_K=R_D\frac{u_e}{u_\infty}\left(\frac{K}{D}\right)\left(\frac{u_K}{u_e}\right) \tag{16}$$

the local magnitude of R_K is dependent upon the body Reynolds number R_D, shape of the body $u_e(s)$, and the roughness height K. In order to examine the sole effect of forebody shape to its sensitivity to roughness, the expression for R_K must be redefined in a more appropriate form.

Assume that the laminar velocity profile varies linearly within the thin wall region defined as $0\leq y\leq K/D$. Thus,

$$\frac{u_K}{u_e}=f'_K\simeq f''_w\eta_K \tag{17}$$

where η_K is defined as

$$\eta_K=\sqrt{R_D\frac{u_e}{u_\infty}\frac{P^2}{(s/D)}\left(\frac{K}{D}\right)} \tag{18}$$

Equation (17) is a very good approximation for laminar boundary layers with favorable pressure gradient and small degrees of roughness. By combining Eqs. (16-18), one obtains

$$\bar{R}_K=(u_e/u_\infty)^{3/2}P(s/D)^{-1/2}f''_w \tag{19}$$

where

$$\bar{R}_K\equiv R_K R_D^{-3/2}(K/D)^{-2} \tag{20}$$

The definition given by Eq. (20) is similar in form to that defined by Tetervin.[16] It is important to note that the dimensionless quantity $\bar{R}_K(s)$ is solely dependent upon the shape of the body for smooth surfaces since f''_w is independent of body Reynolds number. This expression can be used to characterize the sensitivity of a given axisymmetric shape relative to other shapes.

Extensive numerical results comparing the exact expression using Eqs. (16) and (20) with the simplified form given by Eq. (19) show that the values differ by an insignificant amount for $R_K\leq10$. In any event, using Eq. (19) along with the values of f''_w obtained from smooth wall conditions will give a conservative estimate for roughness effects on transition.

The computer results for $\bar{R}_K(s)$ on several body shapes are shown in Fig. 3. These curves allow a relative comparison of the sensitivity to roughness of various elliptical forebody shapes at fixed values of roughness and body Reynolds number. Somewhat similar type results had been presented by Tetervin[16] for a sphere and disk.

Two features of the shape of the curves warrant attention. These are the peak values of $\bar{R}_K$ and the surface location of the maximum value. The results shown in Fig. 3 indicate that slender bodies are very sensitive to roughness tripping the boundary layer in the vicinity of the leading edge or forward stagnation point, while blunt nose bodies (such as a flat nose or disk) will always maintain a laminar flow near the stagnation point but are more sensitive to roughness downstream near the possible location of natural transition.

The critical question as to whether the boundary layer will be tripped is answered by computing the local magnitude of

Table 3 Dimensionless roughness Reynolds number for various body shapes

$R_D\times10^6$	s/D	β	$(K/D)\times10^5$	R_K	$(\nu'_K)_{\max}$	$\frac{(H)_{\mathrm{rough}}}{(H)_{\mathrm{smooth}}}$	$\frac{(f''_w)_{\mathrm{rough}}}{(f''_w)_{\mathrm{smooth}}}$
15	0.657	0.108	0.8150	3.78	0.032	1.001	0.971
38	0.572	0.134	0.4000	4.07	0.037	1.001	0.966
62	0.517	0.155	0.2777	4.40	0.043	1.001	0.961
100	0.485	0.165	0.1920	4.42	0.044	1.001	0.960
50	0.688	0.099	0.5555	9.31	0.182	1.007	0.853
50	0.528	0.152	0.5555	10.88	0.244	1.011	0.815
50	0.332	0.241	0.5555	12.85	0.332	1.019	0.769
50	0.174	0.369	0.5555	12.96	0.337	1.024	0.770

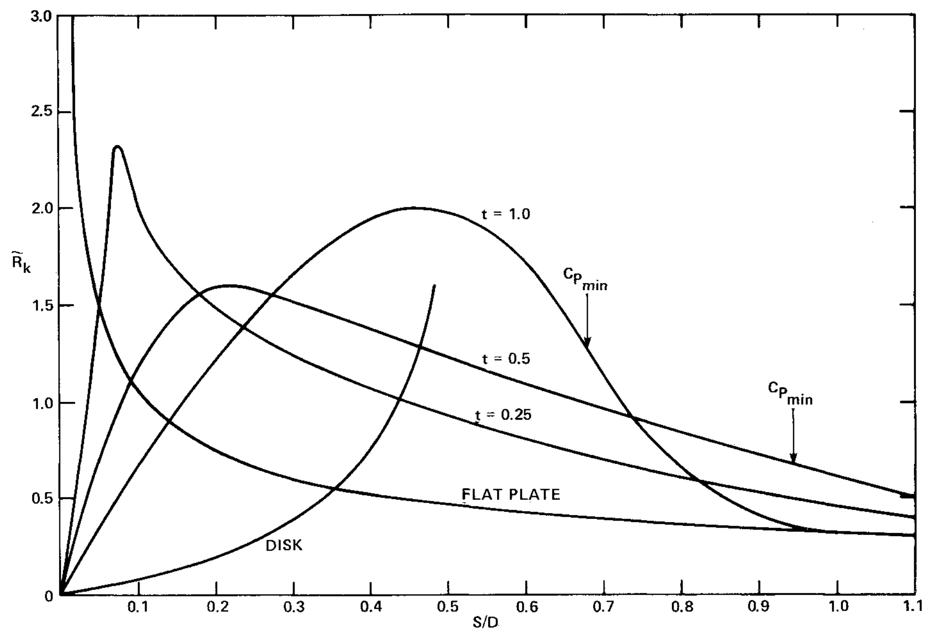

Fig. 3 Dimensionless roughness Reynolds number for various body shapes.

Table 4 Variation of α with elliptical forebody shapes

Shape, t^a	$(x/D)_{inst}$ range	$(s/D)_{inst}$ range	$\bar{R}_K$ range	α range
1.0	0.40-0.50	0.60-0.80	0.60-1.70	1.41-3.65
0.5	0.45-0.80	0.50-1.20	0.40-1.61	1.57-3.16
0.25	0.30-0.75	0.45-0.90	0.50-1.00	1.75-3.16

[a] See Fig. 2. $10\times10^6 \le R_D \le 100\times10^6$.

Table 5 Variation of roughness height with body Reynolds number elliptical forebody ($t = 0.500$), location of roughness element $[(x/D)_K = 0.205]$

R_D	$(\delta^*/D)^a$	$(K/D)_{trip} = \dfrac{4.22}{R_D{}^{3/4}}$	$(K/\delta^*)_{trip}$	Location of natural transition
2.5×10^6	30.23×10^{-5}	6.72×10^{-5}	~0.22	$(x/D)_{tran} \ge 1.00$
25×10^6	9.56×10^{-5}	1.20×10^{-5}	~0.12	$(x/D)_{tran} \approx 0.600$
50×10^6	6.76×10^{-5}	0.711×10^{-5}	~0.10	$(x/D)_{tran} \approx 0.350$
100×10^6	4.78×10^{-5}	0.422×10^{-5}	~0.09	

[a] These values obtained from laminar boundary layer computations at $(x/D)_K = 0.205$.

R_K, given by Eq. (16), along each point on the body surface and determining whether $R_K \ge (R_K)_{crit}$ in the vicinity of the location of natural transition. If so, transition will move upstream of that location at the prescribed roughness and body Reynolds number.

The hemispherical forebody ($t = 1.0$) has a peak value of R_K or $\bar{R}_K$ in the neighborhood of the predicted location for natural transition at Reynolds numbers in the range of $10\times10^6 \le R_D \le 100\times10^6$. Thus, this shape is very sensitive to roughness tripping the boundary layer. Forebody shape identified as $t = 0.50$ appears to be the best shape among the candidates of elliptical forebodies in this range of Reynolds numbers. For this body shape and Reynolds number range, natural transition is predicted to occur in the range $0.45 \le (s/D) \le 0.70$ downstream of the peak value of $\bar{R}_K$ shown in Fig. 3. Furthermore, the peak value of R_K is lower for this body shape, thereby requiring a slightly larger magnitude of roughness prior to tripping.

A practical criterion is required to establish the degree of roughness that will cause the transition to move upstream initially from the location of natural transition. Therefore, assume $(\bar{R}_K)_{crit} \simeq 4.0$ and evaluate Eq. (20) at the location of natural transition

$$\bar{R}_K(s_{inst}) \simeq 4.0 R_D^{-3/2} (K/D)_{crit}^{-2} \tag{21}$$

By defining

$$\alpha^2 \equiv 4.0/[\bar{R}_K(s_{inst})] \tag{22}$$

It follows from Eq. (21) that

$$(K/D)_{crit} \simeq \alpha R_D^{-3/4} \tag{23}$$

Table 4 contains some typical results for a variety of elliptical forebody shapes.

A common approach to measuring the degree of roughness affecting transition is based on the computed value of (K/δ^*). The results from the computations show that the value of (K/δ^*) at the location of natural transition does not vary significantly over a range of Reynolds number. Specifically,

$$(K/\delta^*) \le 0.03 \text{ or } (K/\theta) \le 0.08 \tag{24}$$

are the critical values at the location of natural transition.

The analysis and graphical results presented for uniform roughness in Fig. 3 is equally applicable for localized roughness but is subject to a slightly different interpretation. For this situation, the question raised is "At a prescribed location, what is the value of the localized roughness that will *trip* the downstream laminar flow causing the location of transition to move upstream?"

This is usually the situation when performing laboratory experiments on the effects of various types of roughness tripping the flow. The most widely discussed procedures are based on flat-plate data. A threshold value of (K/δ^*), (K/θ), or R_K is given at the roughness location that will cause the transition location to move upstream, possibly to the location of the roughness element. Smith and Clutter[9] propose that a "safe region" of roughness is selected as $(R_K)_{trip} \leq 25$. Note that this value is much larger than the critical value for distributed roughness.

Accepting the data presented by Smith and Clutter on the flat-plate experiment, one can proceed to calculate the critical roughness height for tripping the laminar flow by also using the results shown in Fig. 3. This procedure requires that at the dimensionless roughness location $(s/D)_K$ one reads the value of $\bar{R}_K$ from Fig. 3. Then setting $R_K = 25$ and solving for (K/D) one obtains from Eq. (20)

$$(K/D)_{trip} \simeq 5[\bar{R}_K(s_K)]^{-1/2} R_D^{-1/4} \equiv \mu R_D^{-1/4} \quad (25)$$

As an example, consider the elliptical forebody $t = 0.500$ with roughness prescribed at the location $x_K/D = 0.205$ or $s_K/D = 0.381$. The problem is to compute the value of roughness that will trip the laminar flow for any prescribed vehicle speed or body Reynolds number. Using Fig. 3 for $t = 0.500$ and $s_K/D = 0.381$, one obtains $\bar{R}_K = 1.40$ thus,

$$(K/D)_{trip} \simeq 4.22 R_D^{-1/4} \quad (26)$$

Table 5 presents a comparison of roughness heights over a range of Reynolds numbers and the predicted location of transition. For this body shape at $R_D = 50 \times 10^6$, natural transition is predicted to occur at $(x/D)_{tran} = 0.350$. These results show that at the roughness location $(x/D) = 0.205$, roughness values of $(K/\delta^*) = 0.10$ will cause transition to occur at that upstream location thereby tripping the flow.

It should be noted that the values of $(K/\delta^*)_{trip}$ are much larger than $(K/\delta^*)_{crit}$ for which the latter reflects the transition location's sensitivity to uniform roughness.

Predicting Onset of Transition

The simplified method for predicting transition will be used to investigate the upstream movement of the location of natural transition on an elliptical forebody due to uniformly distributed roughness.

First, let us consider the case of a smooth surface. The universal curve for $(R_\delta^*)_{crit}$ vs H is the basis for the simplified method. Since the values of the shape factor H are independent of R_D for smooth wall, $(R_\delta^*)_{crit}$ is solely dependent upon the shape of the body. Figure 4 shows the variation of R_δ^* along the surface of a smooth elliptical forebody ($t = 0.500$). The growth of the axisymmetric boundary layer is characterized by the expression

$$R_\delta^* R_D^{-1/2} = \sqrt{(s/D)/(u_e/u_\infty)/P^2} \quad \Delta(\xi,\beta) \quad (27)$$

where

$$\Delta(\xi,\beta) \equiv \int_0^\infty (1-f')\,d\eta \quad (28)$$

Results obtained over a range of Reynolds numbers are shown in Fig. 4. The intersections of the R_δ^* family of curves with the

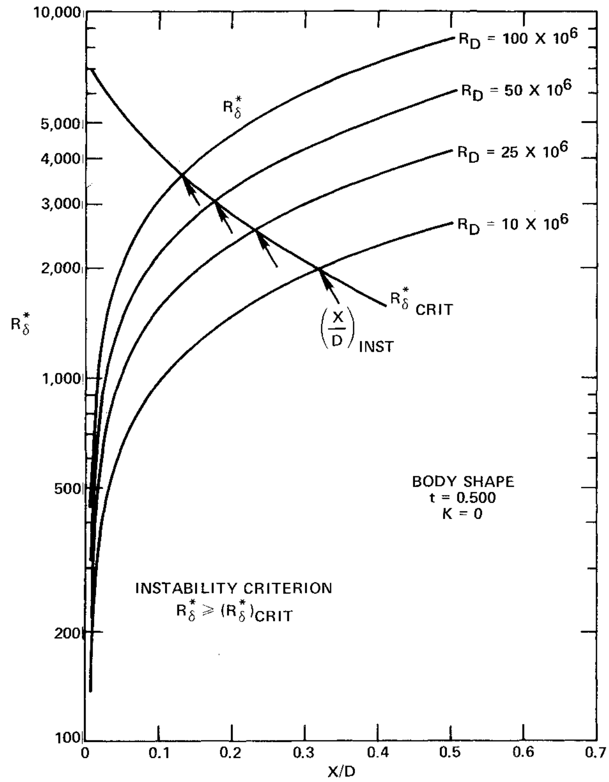

Fig. 4 Effects of body Reynolds number on transition for elliptical forebody shape with smooth surface.

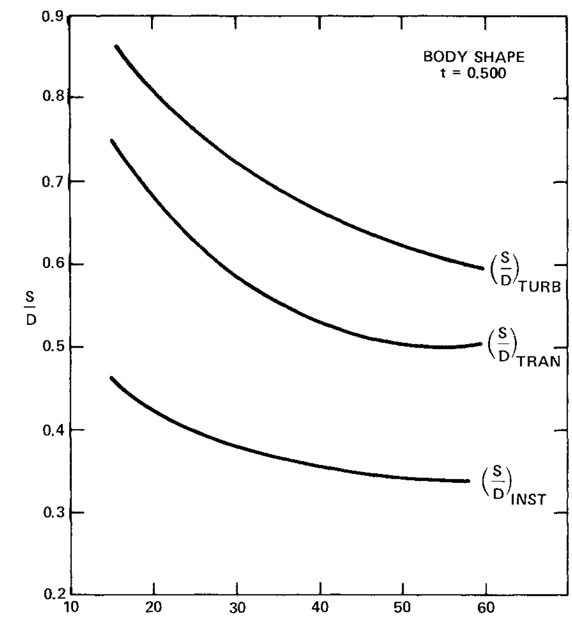

Fig. 5 Predicted bursting regions for $t = 0.500$.

$(R_\delta^*)_{crit}$ define the predicted location for instability.

It is assumed that the position for the onset of burst eruption can be predicted by the empirical relation

$$R_{S_{tran}} \simeq R_{S_{inst}} + 120 R_{S_{inst}}^{2/3} \quad (29)$$

$$\frac{(s_{tran} - s_{inst})}{s_{inst}} \simeq 120 R_{S_{inst}}^{-1/3} \quad (30)$$

This relationship generally predicts that the width of Tollmien-Schlichting wave growth region is larger than the

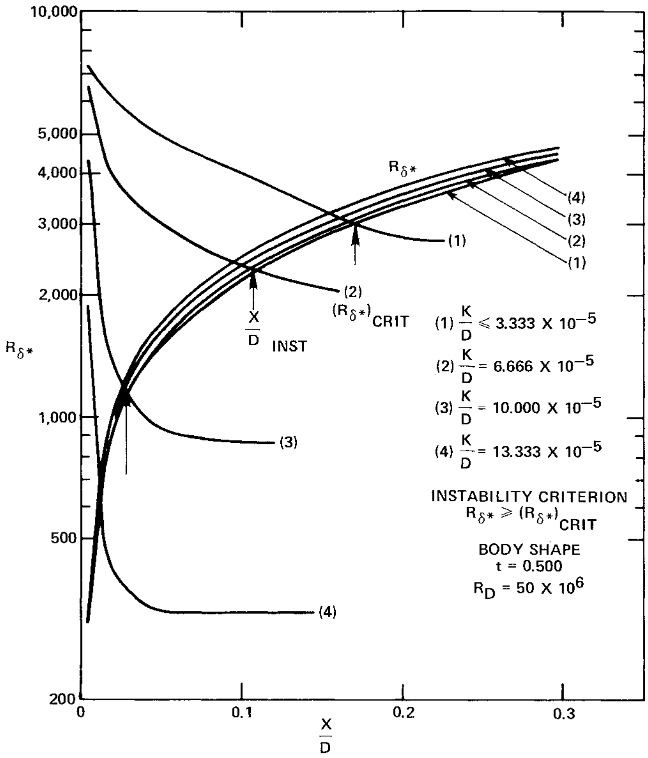

Fig. 6 Effects of roughness height on transition for elliptical forebody shape $t = 0.500$.

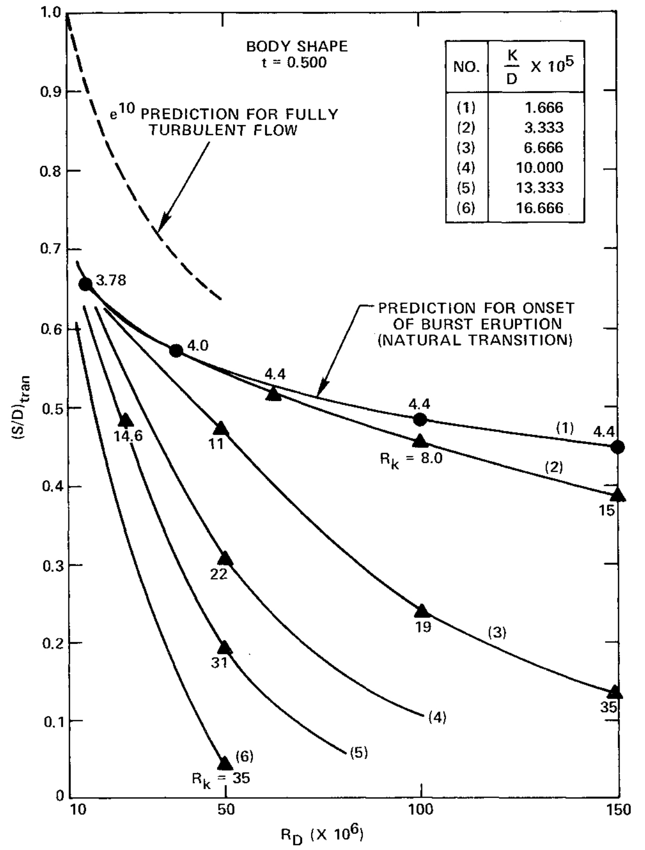

Fig. 7 Variation of transition arc-length with body Reynolds number for elliptical forebody shape $t = 0.500$ with various roughness heights.

width of the bursting region. Figure 5 illustrates the predicted values of the width of transition for a smooth elliptical forebody ($t = 0.500$).

For the computations of roughness in laminar flows, the shape factor H is influenced by the roughness through the roughness viscosity term ν_K, which in turn is dependent upon R_K. Therefore, for rough wall conditions, $(R_\delta^*)_{\rm crit}$ is dependent upon both the degree of roughness and body Reynolds number as well as the body shape.

Figure 6 shows the variation in $(R_\delta^*)_{\rm crit}$ along the body surface at various levels of roughness with a fixed body Reynolds number. Superimposed on Fig. 6 is the R_δ^* family of curves for the boundary-layer growth. As shown in these results, surface roughness does not significantly effect the displacement thickness of the laminar boundary layer exemplified by only slight variation of R_δ^* for the different values of K/D.

The intersection of the R_δ^* family of curves with the $(R_\delta^*)_{\rm crit}$ curves defines the location for instability of the flow with a rough wall condition. Figure 7 is a compilation of the numerical results obtained for the elliptical body showing how the location of transition $(s/D)_{\rm tran}$ varies with body Reynolds number. These results were obtained using the simplified method for predicting instability along with Eq. (29) for extrapolating to transition. A family of transition curves are presented for various degrees of roughness. These results show quantitatively the effects of roughness tripping the laminar flow. Since no experimental data are available at these high Reynolds numbers, the accuracy of this procedure cannot be assessed. However, the order of magnitude of the roughness levels that trip the flow are consistent with some unpublished experimental data for large axisymmetric bodies. The values of R_K at the tripped locations are also indicated in Fig. 7. It is encouraging to see the values of $10 \leq (R)_{\rm trip} \leq 30$ and that these values are consistent with the localized tripping criterion of $(R_K)_{\rm trip} = 25$. Since distributed roughness would be a more severe condition, the results are in surprisingly close agreement with the $(R_K)_{\rm trip} = 25$ suggested by the flat-plate experiments.

Summary and Conclusion

A study was made to develop a method of analysis and computational procedure for predicting the effects of roughness on laminar-turbulent transition. Previous studies by several investigators on both the basic mechanisms for roughness tripping the flow and on an analytical model of a "rough wall" laminar profile were used as the basis for the procedure formulated.

A dimensionless roughness Reynolds number $\bar{R}_K$ was defined which allows one to isolate the effects of body shape on causing premature transition. Therefore, a comparative evaluation of the sensitivity of body shape to tripping the flow can be made by examining the behavior of $\bar{R}_K$ for a class of geometrical shapes.

The individual curve for each shape can be used as the basis for the computations of two critical roughness levels. These are

1) $(K/D)_{\rm crit}$—The level of uniform roughness that will cause transition to move initially upstream of the location for natural transition.

2) $(K/D)_{\rm trip}$—The level of localized roughness that will cause the downstream location of natural transition to move upstream, possibly to the location of the roughness element.

The results for elliptical forebodies indicate that the level of roughness is dependent upon the body Reynolds number according to the approximations

$$(K/D)_{\rm crit} \simeq \alpha R_D^{-3/4}$$

where

$$\alpha = \frac{2}{\sqrt{\bar{R}_K(s_{\rm inst})}} = f\,(\text{shape}, R_D) \tag{31}$$

and

$$(K/D)_{\text{trip}} \simeq \mu R_D^{-3/4}$$

where

$$\mu = \frac{5}{\sqrt{R_K(s_K)}} = f\,(\text{shape}) \tag{32}$$

The values of $(K/\delta^*)_{\text{crit}} \lesssim 0.03$ and $(K/\delta^*)_{\text{trip}} \lesssim 0.10$ are computed for these respective conditions.

The prediction of the actual location of transition on a forebody with a prescribed degree of roughness was done using a simplified method. This procedure was based upon the data from similarity-type laminar profiles. The method has been confirmed only for elliptical type forebodies at $10\cdot10^6 \leq R_D \leq 50\cdot10^6$. The more rigorous e^n method is generally preferred to this method since it has been somewhat established as the best available procedure. However, the complexity of the e^n method might dictate a simple procedure during the preliminary design stage and, then, the more accurate e^n method during the final design. Both methods are of questionable accuracy at high Reynolds numbers since no experimental data have ever been obtained to validate the predictions.

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